Before we start ...

THE HIGGS MECHANISM

[See e.g. Cheng, Li. Gauge theories of elementary particle physics]

$\Rightarrow$ $U(1)$ gauge theory of a scalar field Φ (the "abelian Higgs" model)

$$\mathcal{L} = (D_\mu \Phi)^* (D^\mu \Phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V_0(\Phi)$$

with:

$$D_\mu \Phi = \partial_\mu \Phi - i g A_\mu \Phi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$V_0(\Phi) = m^2 \Phi^* \Phi + \lambda (\Phi^* \Phi)^2$$

The Lagrangian is invariant under the U(1) local transformations:

$$\Phi(x) \longrightarrow e^{-i\chi(x)} \Phi(x)$$

$$A_\mu(x) \longrightarrow A_\mu(x) - \frac{1}{g} \partial_\mu \chi(x)$$

(U(1) gauge symmetry)

For $m^2 < 0 \longrightarrow$ Spontaneous Symmetry Breaking (SSB)

The minimum of the potential is at:

$$|\Phi| = \sqrt{\frac{m^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$$

field excitation around the minimum

Goldstone degree of freedom

The field Φ then reads: $\Phi(x) = \frac{1}{\sqrt{2}} (v + h(x))\, e^{i\xi(x)} = \frac{1}{\sqrt{2}} (v + h(x) + i v \xi(x) + \dots$

$\Rightarrow$ $\varepsilon(x)$ does not appear in $V_0(\Phi)$, as this depends on $\Phi^* \Phi$

We can perform a $U(1)$ gauge transformation to completely remove it from the Lagrangian (transforming to Unitary Gauge)

$$\tilde{\Phi}(x) = e^{-i\varepsilon(x)} \Phi(x) = \frac{1}{\sqrt{2}}(v + h(x))$$

$$B_\mu(x) = A_\mu(x) - \frac{1}{g} \partial_\mu \varepsilon(x)$$

In terms of $\tilde{\Phi}$ and B_μ, we have:

$$\mathcal{L} = \frac{1}{2} \left| \partial_\mu h(x) - i g B_\mu (v + h(x)) \right|^2 - \frac{1}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - V_0(h)$$

$$= \frac{1}{2} \partial_\mu h \, \partial^\mu h - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (gv)^2 B_\mu B^\mu + g^2 v \, B_\mu B^\mu h + \frac{g^2}{2} B_\mu B^\mu h^2$$

$$+ \frac{m^2}{2} (v+h)^2 - \frac{\lambda}{4} (v+h)^4$$

$$= \frac{1}{2} (\partial_\mu h)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \underline{\frac{1}{2} (gv)^2 B_\mu B^\mu} \; \underline{- m^2 h^2} + g^2 v \, B_\mu B^\mu h$$

$$+ \frac{g^2}{2} B_\mu B^\mu h^2 - \lambda v h^3 - \frac{\lambda}{4} h^4$$

Massive scalar h with mass $\sqrt{2}\, m = \sqrt{2\lambda}\, v$

Massive gauge boson B_μ with mass gv