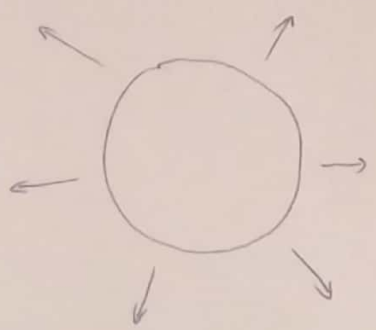
Lecture 5

GRAVITATIONAL WAVES

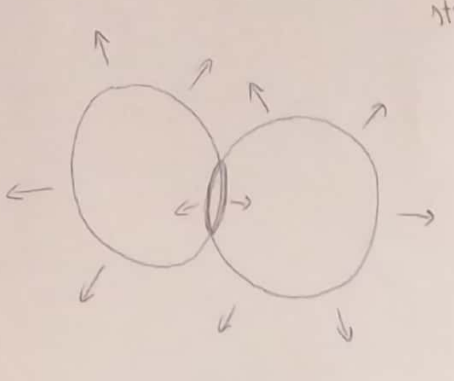
Source of gravitational waves: Transverse-traceless part of energy-momentum tensor T_{ij}^{TT} → "Anisotropic stress"

During the electroweak phase transition (if of first order!), or indeed during any first order phase transition in the early Universe, the process of coalesce of the many nucleated bubbles, via their collisions and subsequent evolution, generates these anisotropic stresses that can source gravitational wave production.

One single expanding bubble is spherically symmetric → no anisotropic stress



Collisions / Coalescence of several bubbles breaks spherical symmetry → anisotropic stress!!



We now derive the (rough) scaling of the gravitational wave signal amplitude from a first order phase transition.

(see e.g. Grojean, Servant, 2006)

Newton's constant

$$P_{GW} \equiv \text{power of gravitational wave emission} \sim G \cdot \left(\dddot{Q}_{ij}^{TT}\right)^2$$

Quadrupole approximation

Where Q_{ij}^{TT} is the quadrupole moment of the source of GW (gravitational waves), which is T_{ij}^{TT}

$$\dddot{Q}^{TT} \sim \frac{\text{Kinetic energy}}{\text{time}} \sim \dot{E}_{kin}$$

Kinetic energy $\sim \frac{\text{mass} \cdot \text{size}^2}{\text{time}^2}$

$$E_{kin} \sim K \cdot \Delta V \cdot R_B^3$$

$K \Delta V = K \, \alpha_N \, \rho_{rad}$

$R_B = V_w \cdot \beta^{-1}$

K: Efficiency factor: How much of the electroweak phase transition energy is available to generate GW

R_B: Bubble radius

Then, $E_{kin} \sim K \cdot \alpha_N \, \rho_{rad} \, V_w^3 \, \beta^{-3}$

$$P_{GW} \sim G \left(\dot{E}_{kin}\right)^2 \sim \frac{H^2}{\rho_{tot}} \frac{1}{\beta^{-2}} \left(K \, \alpha_N \, \rho_{rad} \, V_w^3 \, \beta^{-3}\right)^2$$

ρ_{tot}: $\Delta V + \rho_{rad}$

The energy density in GW, ρ_{GW}, normalized to the total energy density, is

$$E_{GW} = \beta^{-1} P_{GW}$$

$$\Omega_{GW} = \frac{\rho_{GW}}{\rho_{tot}} = \frac{E_{GW}/R_B^3}{\Delta V + \rho_{rad}} = \frac{1}{(1+\alpha_N)\rho_{rad}} \cdot \frac{\beta^{-1} P_{GW}}{R_B^3}$$

$$\Delta V + \rho_{rad} \rightarrow (1+\alpha_N)\rho_{rad}$$

$$= \frac{H^2}{(1+\alpha_N)^2 \rho_{rad}^2} (K\alpha_N)^2 \rho_{rad}^2 \; R_B^3 \; \frac{1}{\beta^{-1}}$$

$$= \frac{H^2}{\beta^2} \left(\frac{K\alpha_N}{1+\alpha_N}\right)^2 V_W^3$$

Recall! (Lecture 3) $\frac{\beta^{-1}}{H^{-1}}$ is the duration of the phase transition in Hubble units

Note that this is Ω_{GW} at the time of gravitational wave production (during the electroweak phase transition). In order to calculate Ω_{GW} today we have to redshift it!

Doing this, and performing a slighty more detailed computation, one arrives at:

$$\Omega_{GW}(f_{peak}) \simeq 10^{-6} \left(\frac{\beta^{-1}}{H^{-1}}\right)^2 \left(\frac{K\alpha_N}{1+\alpha_N}\right)^2 \frac{V_W^3}{0.24 + V_W^3}$$

(Today!)

$$f_{peak} \simeq 10^{-2} \left(\frac{\beta^{-1}}{H^{-1}}\right)^{-1} \frac{T_N}{100\,GeV} \; mHz$$

with $\Omega_{GW}(f_{peak})$ being the maximum ("peak") GW amplitude, and f_{peak} the GW frequency at that amplitude.

As we have seen in Lecture 3, the energy of the electroweak phase transition goes partly into setting the thermal plasma in motion $\left(v(\xi) \right)$ and partly into heating-up the thermal plasma $\left(T(\xi) \right)$

The efficiency coefficient K accounts for the fact that only the part that is converted into kinetic energy of the plasma is then available to source GW production

We can compute K using the hydrodynamic description of the bubble expansion (Lecture 3) for deflagrations and detonations:

(see e.g. Espinosa, Konstandin, No, Servant, 2010)

$$K = \frac{3}{\Delta V \, R_B^3} \int_0^{R_B} T_{rr} \, r^2 \, dr$$

(energy in bulk motion of plasma normalized to vacuum energy of transition)

$$= \frac{3}{\Delta V \, V_w^3} \int w(\xi) \, v^2(\xi) \, \gamma^2(\xi) \, \xi^2 \, d\xi$$

(note that $K < 1$ since the phase transition happens in a thermal plasma and part of the transition energy goes into heating-up the plasma; for a would-be phase transition in vacuum, $K = 1$)

Since K is a function of α_N an V_W, $K(\alpha_N, V_W)$, because those are the parameters determining the hydrodynamic behaviour of the expanding bubbles, then the GW amplitude Ω_{GW} and frequency f_{peak} can be described in therms of the electroweak phase transition parameters!

$T_N,\ \alpha_N,\ V_W,\ \beta^{-1}/H^{-1}$