5 Lecture 4: Final exercise

5.1 What is this lecture about?

In the final lecture we are going to review our previous lectures by going through the whole process for a new model. For this purpose we will choose the model introduced in [24], described in detail in Sec. D. As we will see, this model has a few additional complications that will be helpful to learn a few more features and possibilities of the computer tools presented in this course.

5.2 Implementing the model in `SARAH`

First of all, we must implement the model in `SARAH`. We know already that, in addition to useful analytical results, `SARAH` can also produce input files for the rest of the codes. Therefore, implementing the model in `SARAH` is always a practical approach.

The `SARAH` name of the model will be `DarkBS`. Since most of the definitions are analogous to the ones in lecture 1, we will only highlight those that require further refinements. All `SARAH` model files for the DarkBS model can be found in Appendix G.

DarkBS.m

The model is based on the extended gauge symmetry $U(1)_Y \times SU(2)_L \times SU(3)_c \times U(1)_X$. Although the $\mathbb{Z}_2$ parity obtained after symmetry breaking is automatic, we must tell `SARAH` so that identifies the dark matter candidate,

`DarkBS.m`

```
Global[[1]] = {Z[2], Z2};
```

The definition of the gauge groups must contain an additional piece:

`DarkBS.m`

```
Gauge[[1]]={B,   U[1], hypercharge, g1,False,1};
Gauge[[2]]={WB, SU[2], left,        g2,True,1};
Gauge[[3]]={G,  SU[3], color,       g3,False,1};
Gauge[[4]]={Bp,  U[1], Uchi,        gX,False,1};
```

The additional gauge group must also appear in the definition of the particles in the model. For example, the vector-like fermions are defined as

`DarkBS.m`

```
FermionFields[[6]] = {lL, 1, {v4, e4},    -1/2, 2,  1,  2, 1};
FermionFields[[7]] = {lR, 1, {e5, v5},     1/2, 2,  1, -2, 1};
FermionFields[[8]] = {qL, 1, {u4, d4},     1/6, 2,  3,  2, 1};
FermionFields[[9]] = {qR, 1, {d5, u5},    -1/6, 2, -3, -2, 1};
```

Notice that all right-handed fields have opposite gauge charges to those for the left-handed ones. This is equivalent to identifying, for example, `lR` with $\overline{L_R}$. As a consequence of this, we must write the components of the right-handed doublets as in a `-2` of $U(1)_X$. This practical choice simplifies the writing of the Lagrangian, which takes a more transparent form. For instance, for the Yukawa terms one has

`DarkBS.m`

```
LagHC =  -(Yd conj[H].d.q + Ye conj[H].e.l + Yu H.u.q + mQ qL.qR + mL ↪
    lL.lR + lamQ Phi.qR.q + lamL Phi.lR.l );
```

The additional $U(1)_X$ factor implies the existence of an additional neutral gauge boson. Since this vector will get a mass after the spontaneous breaking of the gauge symmetry, we can identify it with the Z' boson. This is important when defining the gauge sector:

`DarkBS.m`

```
DEFINITION[EWSB][GaugeSector] =
{
  {{VB,VWB[3],VBp},{VP,VZ,VZp},ZZ},
  {{VWB[1],VWB[2]},{VWp,conj[VWp]},ZW}
};
```

The rest of the `DarkBS.m` file goes along the same lines of the `Scotogenic.m` file described in the first lecture. The only detail that should be pointed out is the resulting fermion mixings,

DarkBS.m

```
{{{dL,d4}, {conj[dR],d5}}, {{DL,Vd}, {DR,Ud}}},
{{{uL,u4}, {conj[uR],u5}}, {{UL,Vu}, {UR,Uu}}},
{{{eL,e4}, {conj[eR],e5}}, {{EL,Ve}, {ER,Ue}}},
```

It is important to note that the field that should be written in the basis definition for the charged leptons is `e5`, and not `conj[e5]`. One can easily understand this fact by having a look at the way `eR` and `e5` are defined in the Matter Fields section.

parameters.m

$U(1)$ mixing is a general feature in models with several $U(1)$ factors. `SARAH` can perfectly handle this property, but we must define the mixed gauge couplings in the `parameters.m` file

parameters.m

```
{g1X,        {LaTeX -> "\\tilde{g}",
              LesHouches -> {GAUGE,10},
              OutputName -> g1X}},
{gX1,        {LaTeX -> "\\bar{g}",
              LesHouches -> {GAUGE,11},
              OutputName -> gX1}},
```

For the mixing matrices of the CP-even and CP-odd neutral scalars we have to add the lines

parameters.m

```
{ZH, { Description->"Scalar-Mixing-Matrix",
       LaTeX -> "Z^H",
       Real -> True,
       DependenceOptional ->   {{-Sin[\[Alpha]],Cos[\[Alpha]]},
                                 {Cos[\[Alpha]],Sin[\[Alpha]]}},
       Value -> None,
       LesHouches -> SCALARMIX,
       OutputName-> ZH     }},

{ZA, { Description->"Pseudo-Scalar-Mixing-Matrix",
       LaTeX -> "Z^A",
       Real -> True,
       DependenceOptional -> {{-Cos[\[Beta]],Sin[\[Beta]]},
                               {Sin[\[Beta]],Cos[\[Beta]]}},
       Value -> None,
       LesHouches -> PSEUDOSCALARMIX,
       OutputName-> ZA      }},
```

The inclusion of the `Description` options is crucial. This is because it is necessary to properly identify these two matrices since they play a role in some specific calculations (for example, the calculation of the Higgs boson flavor violating decay rate to a pair of leptons, $h \to \ell_i^+ \ell_j^-$). Without this, `SARAH` would not know how to identify these matrices among all the mixing matrices in the model. Notice also that these two mixing matrices have been expressed in terms of the angles α and β.

Finally, the mixing matrix in the neutral gauge sector is also defined in terms of two angles: θ_W and θ'_W:

parameters.m

```
{ZZ, { Description -> "Photon-Z Mixing Matrix",
       Dependence->  {{Cos[ThetaW],-Sin[ThetaW] Cos[ThetaWp], Sin[ThetaW] ↪
           Sin[ThetaWp]},
                      {Sin[ThetaW],Cos[ThetaW] Cos[ThetaWp],-Cos[ThetaW] ↪
                          Sin[ThetaWp]},
                      {0, Sin[ThetaWp], Cos[ThetaWp]}} }},
```

particles.m

There are just a few details worth pointing out in the `particles.m` file. They all have to do with the same feature in this model: many existing sets of mass eigenstates are now extended to included additional particles. For example, the model has two CP-even neutral scalars,

particles.m

```
{hh   ,  {  Description -> "Higgs",
            PDG -> {25,35},
            PDG.IX -> {101000001,101000002} }},
```

and, for example, four charged leptons,

particles.m

```
{Fe,    { Description -> "Leptons",
          PDG -> {11,13,15,17},
          PDG.IX -> {-110000601,-110000602,-110000603,-110000604} }},
```

It is also very important to define the new Z' boson. This is done with the lines

particles.m

```
  {VZp,    { Description -> "Z'-Boson",
             Goldstone -> Ah[{2}] }},
```

Notice that the option `Description` has been used to take advantage of the general definition of Z' bosons in the file `$PATH/SARAH-X.Y.Z/Models/particles.m`. Moreover, we must indicate the Goldstone boson that constitutes the longitudinal part of the massive Z'. In this case this is given by the second CP-odd neutral scalar, the first one being the Goldstone boson of the SM Z boson.

SPheno.m

Finally, the last model file is `SPheno.m`. We have decided to use again a low scale version of `SPheno`. There are only two details which differ slightly from the `SPheno.m` file we prepared for the scotogenic model. Let us comment on them.

The first comment is about the tadpole equations. In this model there are two scalar fields acquiring a VEV. Therefore, we must solve two tadpole equations and hence select two parameters to solve them:

SPheno.m

```
  ParametersToSolveTadpoles = {mH2, mPhi2};
```

The second comment is about a feature that we can exploit to make our life simpler when we are targeting a specific value of a derived parameter. `SPheno` must have input values for all the parameters of the model in order to run properly. However, we can choose between giving this input *directly* or *indirectly*. In the first case, we introduce the values for the fundamental parameters in the Lagrangian. In the second, we introduce the values for some derived parameters which do not appear directly in the Lagrangian, like a gauge boson mass, and tell `SPheno` (and `SARAH`) how to obtain the fundamental parameters from them. This is useful when we are interested in a parameter point with a specific value for a derived parameter.

In the model under consideration, the Z' mass is an important derived parameter, since the phenomenology strongly depends on its precise value. It depends on two quantities, the new gauge coupling g_X and the $SU(2)_X$ breaking VEV, v_ϕ, via

$$m_{Z'} = 2\, g_X\, v_\phi \,. \tag{26}$$

Given the relevance of the Z' mass, it is useful to replace v_ϕ by $m_{Z'}$ as input parameter. We begin by introducing the Z' mass as one of the input parameters in the `MINPAR` block,

SPheno.m

```
  {20, gXInput},
  {21, MZpMass}
```

And then, among the definitions in `BoundaryLowScaleInput` we establish the relation with v_ϕ, SPheno.m

```
{gX,  gXInput},
{g1X, 0},
{gX1, 0},
{vP,  MZpMass/(2*gX)}
```

This way `SPheno` will have input values for all the relevant parameters of the model and we will make sure that $m_{Z'}$ has exactly the value we are interested in. Note also that we have considered a scenario with vanishing $U(1)$ mixing by setting the mixed gauge couplings to zero.

5.3 Generating input files for the other tools

Once the model is implemented in `SARAH` we can generate input files for the other tools. Instead of generating input for the different tools one by one, we can make use of the `MakeAll[]` command to generate input files for all the tools at once. This will generate automatically the `SPheno` module as well as input files for `MicrOmegas` and `MadGraph`. Therefore, we just have to execute the following three lines in `Mathematica`:

```
<<$PATH/SARAH-X.Y.Z/SARAH.m;
Start["DarkBS"];
MakeAll[]
```

The results will be saved in different sub-folders of the `$PATH/SARAH-X.Y.Z/Output/DarkBS/EWSB` folder. Notice that `MakeAll[]` also includes the generation of the LaTeX files with all the model details.

5.4 Benchmark point and numerical results

The first thing we can do after executing `MakeAll[]` is to compile our new `SPheno` code. This step was explained in Sec. 2.6 and the process in this case is completely analogous:

```
$ cd $PATH/SPheno–X.Y.Z
$ mkdir DarkBS
$ cp $PATH/SARAH–X.Y.Z/Output/DarkBS/EWSB/SPheno/* ↪
    $PATH/SPheno–X.Y.Z/DarkBS
$ make Model=DarkBS
```

Next, let us consider a benchmark point for the DarkBS model. We will call it benchmark point **BDarkBS1** (Benchmark DarkBS 1), and it is defined by the input parameters

$$\begin{array}{lll}
\lambda = 0.25 & \lambda_\phi = 0.1 & \lambda_\chi = 10^{-5} \\
\lambda_{\phi\chi} = 10^{-5} & \lambda_{H\phi} = 0 & \lambda_{H\chi} = 0 \\
m_\chi^2 = 3 \cdot 10^6 \, \mathrm{GeV}^2 & m_Q = 1 \, \mathrm{TeV} & m_L = 1 \, \mathrm{TeV} \\
g_X = 1 & m_{Z'} = 4 \, \mathrm{TeV} &
\end{array}$$

$$\lambda_Q = \begin{pmatrix} 0 \\ 3 \cdot 10^{-3} \\ 3 \cdot 10^{-3} \end{pmatrix} \qquad \lambda_L = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

In order to use this parameter point we must include the following lines in the `LesHouches.in.DarkBS` input file:

LesHouches.in.DarkBS

```
Block MINPAR       # Input parameters
1   0.250000E+00    # LambdaInput
2   0.100000E+00     # LPInput
3   0.000010E+00     # LCInput
```

```
4    0.000010E+00     # LCPInput
5    0.000000E+00     # LHPInput
6    0.000000E+00     # LHCInput
10    3.000000E+06    # mChi2Input
11    1.000000E+03    # mQInput
12    1.000000E+03    # mLInput
20    1.000000E+00    # gXInput
21    4.000000E+03    # MZpMass
Block LAMQIN #
1 0.0 #
2 3.0E-3 #
3 3.0E-3 #
Block LAMLIN #
1 0.0 #
2 1.0 #
3 0.0 #
```

By running `SPheno` we can see that the **BDarkBS1** benchmark point has a Higgs mass consistent with the observed value by ATLAS and CMS. Moreover, the `MASS` block also shows that the light neutrinos are massless in this model, whereas the two heavy neutral leptons form a Dirac pair,

`SPheno.spc.DarkBS`

```
          12      0.00000000E+00   # Fv_1
          14      0.00000000E+00   # Fv_2
          16      0.00000000E+00   # Fv_3
          18     -1.73205081E+03   # Fv_4
          20      1.73205081E+03   # Fv_5
```

5.5 Calculating the dark matter relic density

As the next step in our phenomenological study, we can compute the dark matter relic density in the **BDarkBS1** benchmark point using `MicrOmegas`. As explained in Appendix D, the spontaneous breaking of the continuous $U(1)_X$ gauge symmetry leaves a remnant $\mathbb{Z}_2$ that stabilizes the χ scalar. This is therefore the dark matter particle in this model.

The calculation of the dark matter relic density is straightforward and follows the same procedure as for the scotogenic model. Using the files in the `$PATH/SARAH-X.Y.Z/Output/DarkBS/EWSB/CHep` folder, we can proceed in exactly the same way. We find $\Omega_{\rm DM} h^2 = 0.132$, in reasonable agreement with the observed value. The most important annihilation channels are $\chi\,\chi \to d_4\,\bar{d}_4$ and $\chi\,\chi \to u_4\,\bar{u}_4$, this is, to final states including heavy vector-like quarks.

5.6 Signatures at the LHC

Finally, we can use `MadGraph` for some simple (but illustrative) LHC simulations. This model has a Z' boson with a relatively large branching ratio into a pair of muons, $\mu^+\mu^-$. Therefore, let us consider

$$p\,p \to \mu^+\,\mu^-$$

at the LHC. This process will receive many different contributions. Among them, the one induced by s-channel Z' exchange, $p\,p \to Z' \to \mu^+\,\mu^-$. However, note that in the benchmark point **BDarkBS1** the Z' production at the LHC is strongly suppressed, since we have taken $\lambda_Q^1 = 0$. Given that protons have very little content of second and third family quarks, this leads to tiny production cross-sections for the Z'. Moreover, we had a Z' mass of 4 TeV, which again suppresses its production. Therefore, let us consider a new benchmark point, called **BDarkBS2** (Benchmark DarkBS 2), where these are changed. The only changes with respect to the **BDarkBS1** point are:

$$m_{Z'} = 300\,\mathrm{GeV} \qquad \lambda_Q = \begin{pmatrix} 1 \\ 3\cdot 10^{-3} \\ 3\cdot 10^{-3} \end{pmatrix}$$

This parameter point is of course experimentally excluded, since such a light and strongly coupled Z' boson would have been observed already at the LHC. However, it serves as an academic illustration. In order to use this parameter point we must modify some lines in the `MINPAR` and `LAMQIN` blocks of the `LesHouches.in.DarkBS` input file:

`LesHouches.in.DarkBS`

```
Block MINPAR      # Input parameters
1   0.250000E+00   # LambdaInput
2   0.100000E+00    # LPInput
3   0.000010E+00    # LCInput
4   0.000010E+00    # LCPInput
5   0.000000E+00    # LHPInput
6   0.000000E+00    # LHCInput
10   3.000000E+06   # mChi2Input
11   1.000000E+03   # mQInput
12   1.000000E+03   # mLInput
20   1.000000E+00   # gXInput
21   3.000000E+02   # MZpMass
Block LAMQIN #
1 1.0 #
2 3.0E-3 #
3 3.0E-3 #
```

After running `SPheno` with this point we generate a new `SPheno.spc.DarkBS` file that we can now use with `MadGraph`. We will follow the same procedure as for the scotogenic model:

```
$ cd ..
$ $ bin/mg5_aMC
MG5_aMC > import model DarkBS −modelname
MG5_aMC > define p d1 d1bar d2 d2bar u1 u1bar u2 u2bar g
MG5_aMC > generate p p > e2 e2bar
MG5_aMC > output SimDBS
MG5_aMC > launch SimDBS
```

In this simulation we will also include hadronization, showering and detector response effects. This requires running `Pythia` and `PGS`. Therefore, we will answer `2` to the first question, which will require further confirmation by pressing enter. To the second question we will answer `2` in order to modify the `run_card.dat` file. Instead of 10^4 events, we want to generate 10^5. This will imply a more precise simulation. In order to increase the number of events we just have to modify the option `nevents`, which now will read

`run_card.dat`

```
  100000 = nevents ! Number of unweighted events requested
```

We should also modify the `param_card.dat` file before we save and close the `run_card.dat` file. Again, we will simply copy the file we generated with `SPheno` for the **BDarkBS2** benchmark point,

```
$ cp $PATH/SPheno−X.Y.Z/SPheno.spc.DarkBS ↪
    $PATH/MG5_aMC_vX/SimDBS/Cards/param_card.dat
```

Then we are ready to save and close the `run_card.dat` file and press enter in `MadGraph`. The simulation starts, running the standard simulation at parton level, hadronization and detector response. After some minutes (remember, we are using 10^5 random events) it will be finished, showing a cross-section of about 831 pb. We will now use `MadAnalysis` to plot the results. This powerful tool has many features for the analysis of the `MadGraph` results. One can in principle use it in a similar way as `MadGraph`, opening the code and executing some commands one by one in the therminal. Instead, in this case we will use an external file with teh collection of commands we want `MadAnalysis` to execute. This file can be placed in the `MadAnalysis` folder:

plotDarkBS.txt

```
import $PATH/MG5_aMC_vX/SimDBS/Events/run_01/tag_1_pgs_events.lhco.gz
plot M(mu+ mu-) 100 50 500 [logX logY]
submit DarkBSPlot
```

As usual, `X` must be replaced in the first command by the specific `MadGraph` version we are using. With these commands, we are just telling `MadAnalysis` to import the results of our simulation and to an histogram with the number of $\mu^+ \mu^-$ events as a function of the invariant mass of the muon pair, $m_{\mu\mu}$. We have also decided to use logarithmic scales in both axes, and show the invariant mass between 50 GeV and 500 GeV distributed in 100 bins. Finally, the result of the analysis should be saved in a folder called `DarkBSPlot`.

Therefore, we just have to call `MadAnalysis` with this input file

```
$ cd $PATH/madanalysis5
$ bin/ma5 –R plotDarkBS.txt
```

The flag `-R` is used because we are going to read a dataset in lhco format, the one used by `PGS`. If we wanted to use the dataset without hadronization and detector response, instead of importing the file `tag_1_pgs_events.lhco.gz` we would have to import `unweighted_events.lhe.gz`, in lhe format, which would not require the `-R` flag.

`MadAnalysis` will ask us the number of cores we want to use for the analysis. After answering the question it will proceed to the compilation of some parts of the code and the analysis of our results. Eventually, it will be finished and the folder `$PATH/madanalysis5/DarkBSPlot` will be created. In this folder we can find the results in several formats. We can open a pdf file where these are nicely presented using the `MadAnalysis` command

```
ma5 > open DarkBSPlot/PDF
```

In the last page of this pdf file we can see the histogram generated by `MadAnalysis`. There are two visible peaks, at $m_{\mu\mu} = 91$ GeV and $m_{\mu\mu} = 300$ GeV. We have *discovered* two particles: the Z and Z' bosons.

5.7 Summary of the lecture

We conclude the course here. In the last lecture we have reviewed our previous lectures by applying what we have learned to a new model with a few additional complications. After implementing the model in `SARAH`, we have used `SPheno` to obtain numerical results for a specific benchmark point, `MicrOmegas` to compute the relic density and `MadGraph` to run a simple but interesting collider study.